Dynamic programming approaches for estimating and applying large-scale discrete choice models

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People go through their life making all kinds of decisions, and some of these decisions affect their demand for transportation, for example, their choices of where to live and where to work, how and when to travel and which route to take. Transport related choices are typically time dependent and characterized by large number of alternatives that can be spatially correlated. This dissertation deals with models that can be used to analyze and predict choices in large networks. The models and methods proposed in this dissertation are highly relevant for, but not limited to, transport applications.

The dissertation consists of seven articles containing a number of models and methods for estimating, applying and testing large-scale discrete choice models. The contributions of the dissertation can be grouped into three themes: route choice modeling, large-scale multivariate extreme value (MEV) model estimation and nonlinear optimization algorithms.

In fives articles we focus on the problem of choosing paths in a transport network (also known as the route choice problem). The route choice problem deals with identifying which route would be taken by a traveler to go from a specific location to another one in a transport network. Route choice models are important in many transportation applications. They can be used to predict path choices but also to assess travelers’ preferences of various route characteristics (e.g. travel time, travel cost) as well as the effect of travelers’ characteristics to the choice of route (e.g. age, sex, income, gender). In this context, discrete choice models are generally used with revealed preference data for analyzing path choices in real networks.

There are two main challenges associated with discrete choice models for route choice analysis. First, choice sets of paths are unknown to the analyst and the sets of all feasible paths between an origin-destination pair (also known as universal choice sets) cannot be enumerated. Second, path utilities may be highly correlated, for instance, due to physical overlap in the network. It is important to properly model correlated utilities in order to obtain accurate forecasts. Many studies in the literature take an approach that consists of sampling paths from the network and estimate models based on these samples. This approach can be difficult in practice for both estimation and prediction, especially when alternatives are believed to be correlated. In the

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dissertation we develop models based on the dynamic discrete choice framework (also known as parametric Markov decision processes) (Rust, 1987) that allows to model the choice of path in a network without sampling any choice sets. The idea of using the dynamic discrete choice framework for the route choice problem was originally proposed by Fosgerau et al. (2013). In this study, they present a recursive logit (RL) model that can be consistently estimated and quickly used for prediction (predicting traffic flows and generating path choices) without any sampled choice set. The model is however equivalent to the multinomial logit (MNL), hence exhibiting the independence of irrelevant alternatives (IIA) property. This property does not hold when to path overlap in the network. Moreover, relaxing the IIA is challenging because it leads to complicated dynamic programming problems that are very expensive to solve when searching over the parameter space. Our objectives are therefore to develop route choice models that can keep the advantages of the RL model while allowing to conveniently capture the correlation between path utilities in the network and relax the IIA property.

The dissertation contributes to route choice modeling by proposing four different approaches to relax the IIA property of the RL model. First, we propose a nested recursive logit (NRL) model that allows the scales of the random terms to be link specific. We show that the NRL model relaxes the IIA property and allows path utilities to be correlated in a fashion similar to nested logit (Ben-Akiva, 1973). Second, we design a generalized recursive model (called recursive network multivariate extreme value model, RNMEV) that allows the random terms at each choice stage to be correlated by any member of the family of network MEV models (Daly and Bierlaire, 2006). The third approach is motivated by the fact that the mixed logit model is attractive and fully flexible, in the sense that it can approximate any random utility model (McFadden and Train, 2000). We therefore develop an Error Component (EC) model that can be used with mixed RL models to capture the correlation between path alternatives that share subnetwork components. In the last approach we investigate the use of the minimum random regret (RRM) decision rule (Chorus, 2010) to relax the IIA property. We extend an existing RRM model and also develop new nonlinear-in-parameters random utility maximization (RUM) models.

Even though the proposed models allow to conveniently capture the correlation between path utilities, the key challenge lies in their estimation, as it requires solving many dynamic programming problems (value functions) to evaluate the log-likelihood function. We therefore propose solution methods that allow to estimate the proposed recursive models in short computational time, so that they become more attractive in practice for both estimation and prediction. More precisely, we develop a decomposition method to speed up the estimation of the RL model, thus opening the possibility to estimate the mixed RL and nonlinear-in-parameters RRM/RUM models. For the NRL model, we propose a value iteration method with dynamic accuracy to quickly solve the value functions, and for the RNMEV model we show how to simplify the resulting dynamic programming problems by integrating networks of correlation structures into the transport network. We use these methods to estimate the proposed models with large networks using real data and we also perform cross-validation studies.
Besides the aforementioned route choice models and estimation methods, we present an article related to a statistical test for route choice models. More precisely, we show how the information matrix test proposed by White (1982) can be applied to test MNL based route choice models. The results on real data imply that model structures with correlated random terms, such as the NRL, mixed MNL or RNMEV models proposed in this dissertation should be investigated.

We note that the models and methods presented in this dissertation are applied to uni-model networks where the link attributes are assumed to be static and deterministic. These models can be easily used for dynamic and deterministic networks. If the link attributes are stochastic, the corresponding recursive models become more difficult to solve, as the Markov transition probabilities are no longer degenerate. Ongoing work related to such networks is represented in more detail in the dissertation.

In addition to the route choice modeling contributions, we present one article related to the estimation problem of large-scale MEV models. We show that a modeling idea from recursive models can be used to reformulate and estimate large-scale MEV models with network-based correlation structures. Such networks are rooted and directed and a node without successor represents an alternative. These networks are different from transport networks in the sense that there are multiple destinations for each observation, and the link utilities are defined based on the structural parameters of the MEV models. We formulate MEV models as dynamic discrete choice models on the networks of correlation structures, in which the random terms have different scales. We then develop an estimation method based on the concept of network flows and the nested fixed point (NFXP) algorithm (Rust, 1987). We also show that these dynamic models are consistent with the McFadden’s MEV theory and generalize the network MEV model. The proposed dynamic technique can therefore have an impact in any large-scale discrete choice application because it is general and allows to estimate parameters and compute predicted choice probabilities in short computational time.

The seventh article is related to nonlinear optimization algorithms designed from maximum likelihood estimation problems. The objective function becomes costly to evaluate for the models considered in this dissertation. In this case it is particularly important to converge in as few iterations as possible. We consider structured quasi-Newton techniques (Dennis, Jr. and Schnabel, 1996) and focus on the impact of Hessian approximation methods on the performance of line search and trust region algorithms (e.g. Nocedal and Wright, 2006). We adapt and develop methods that allow to switch between different Hessian approximations at each iteration of the optimization algorithm. These methods can be integrated into optimization algorithms to improve their performance. We assess numerical efficiency of the proposed switching methods using mixed logit and a nonlinear-in-parameters recursive model. The algorithms have also been used to estimate other models presented in the dissertation. Moreover, the proposed algorithms are not limited to discrete choice applications, as they may be useful in other nonlinear estimation problems.

The contributions of this dissertation can have an impact in various route choice and discrete
choice applications. The estimation algorithms are implemented in MATLAB and we share the code freely via Git-hub as open source projects (https://github.com/maitien86/). This code can be adapted to deal with other network-based choice problems, e.g. sequences of daily activities or sequences modes in public transport networks. Moreover, in the era of big data analytics, models that can deal with large choice sets and sequential choices are important. Our research can be of interest in various demand analysis applications (predictive analytics) or can be integrated with optimization models (prescriptive analytics). Furthermore, our studies indicate the potential of dynamic programming techniques in this context, even for static models, which opens up a variety of future research directions.

References


